

Month 2 – Trigonometry

Introduction

Welcome to the second month of Trigonometry. I hope you are getting very familiar with the basics that we discussed in the first month. We are building on previous knowledge all the time.

Last week we started graphing by looking at the sine and cosine in great detail. This month we will complete our graphing with the remaining four functions as well as a study of inverse functions. Understanding the graphing of the six functions helps us to understand the inverse functions and why we need them. Their role is critical in solving trigonometric equations and more advanced topics. We will also add to our list of identities as we simplify and verify them.

Let's get started. Don't forget you can contact me and ask questions at any time.

Week 5 – More on Graphing and Inverse Trigonometric Functions

Overview

This week we hope to:

- Become familiar with graphing the tangent function.
- Be able to recognize the reciprocal function graphs.
- Learn how to use inverse functions.

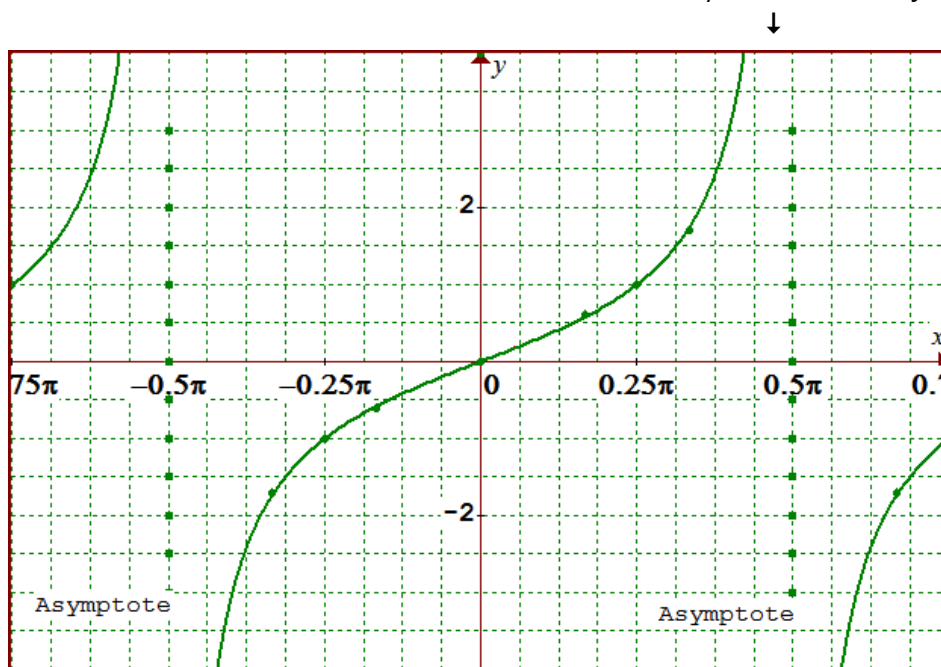
Algebra knowledge needed:

- Asymptote and mathematical notation
- Idea of inverse function
- Interval notation

Graphing the Tangent Function in the Form $y = a \tan(bx+c) + d$

Last month you had an introductory look at the tangent function graph. Let's look at that again and point out the characteristics.

As $x \rightarrow \pi/2$ from the left, $y \rightarrow \infty$.



As $x \rightarrow -\pi/2$ from the right, $y \rightarrow -\infty$.

The properties or characteristics we found:

- The **domain** is all real numbers $x \neq \frac{\pi}{2} + k\pi$, where k is an integer.
- The **range** is the set of all real numbers or the interval $(-\infty, +\infty)$.
- The **period** is π .
- **Vertical asymptotes** appear at the beginning and end of the period at $x \neq \frac{\pi}{2} + k\pi$, where k is an integer.

Other observations:

- From $x = -\frac{\pi}{2}$ to $\frac{\pi}{2}$, which contains the origin, is the pattern for the tangent function.
- The tangent is not defined at multiples of $\frac{\pi}{2}$ because the cosine is 0 at those values of x (Remember the quotient identity $\tan x = \frac{\sin x}{\cos x}$?).
- The behavior at those undefined values of x is significant. The vertical lines are asymptotes, since as x gets closer and closer to those values (we say “ x approaches”), the y -values approach $\pm\infty$ depending on which side of the x -value you look.
- **The x -intercept occurs where $x = 0$ in the center of the period** or cycle.
- There **is no amplitude** for this function as a result of the behavior near undefined values.
- Points that are one-fourth and three-fourths of the way through the cycle are found with $\tan \frac{\pi}{4} = 1$ and $\tan \left(-\frac{\pi}{4}\right) = -1$. (Remember the **special angle** 45° or $\frac{\pi}{4}$?)

Therefore, the characteristics we will use are the period, middle point of the cycle, one-fourth and three-fourths of the cycle, and the vertical asymptotes for each repeating period. The same transformations and translations may occur.

Example 1: Find the period, vertical translation, and equations for the asymptotes for $y = \tan(.5x - \pi) + 1$. Then graph for at least one cycle.

The **period** is $\frac{\pi}{b} = \frac{\pi}{.5} = 2\pi$. Notice that **we must use the period π for the basic tangent** in this formula, not the 2π which was the period for the sine and cosine.

The **vertical displacement** is up 1 unit, so $y = 1$ becomes the new x-axis.

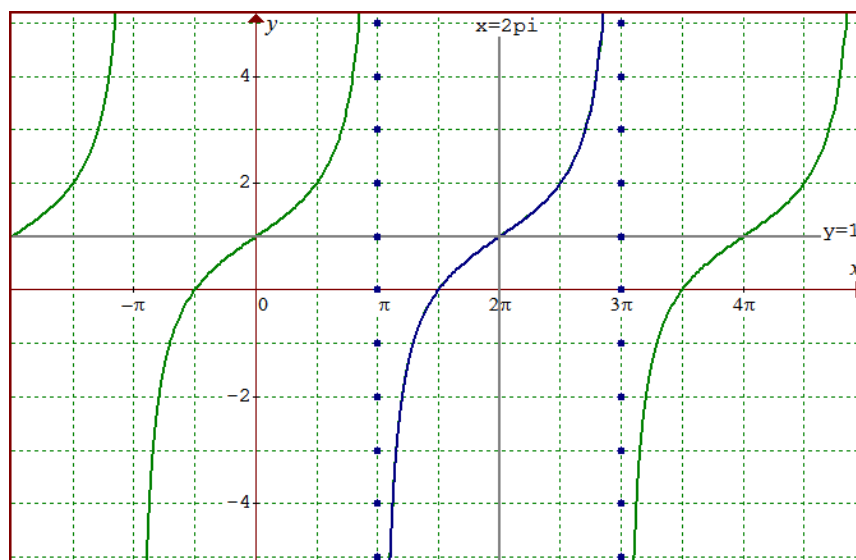
To find the **center of the cycle**, which for the basic graph is at $x = 0$, we compare $(.5x - \pi)$ to 0. Solving $.5x - \pi = 0$ yields $x = 2\pi$ as the new center, with the central x-intercept on the new x-axis.

To find the **starting point of the period** is the same as finding the left vertical asymptote. Since the basic tangent started at $x = -\frac{\pi}{2}$, we compare $(.5x - \pi)$ to $-\frac{\pi}{2}$. Solving the equation $.5x - \pi = -\frac{\pi}{2} \Rightarrow x - 2\pi = -\pi \Rightarrow x = \pi$ is the left asymptote.

If we go 2π units (one period) to the right from that, we get the right side asymptote $x = 3\pi$.

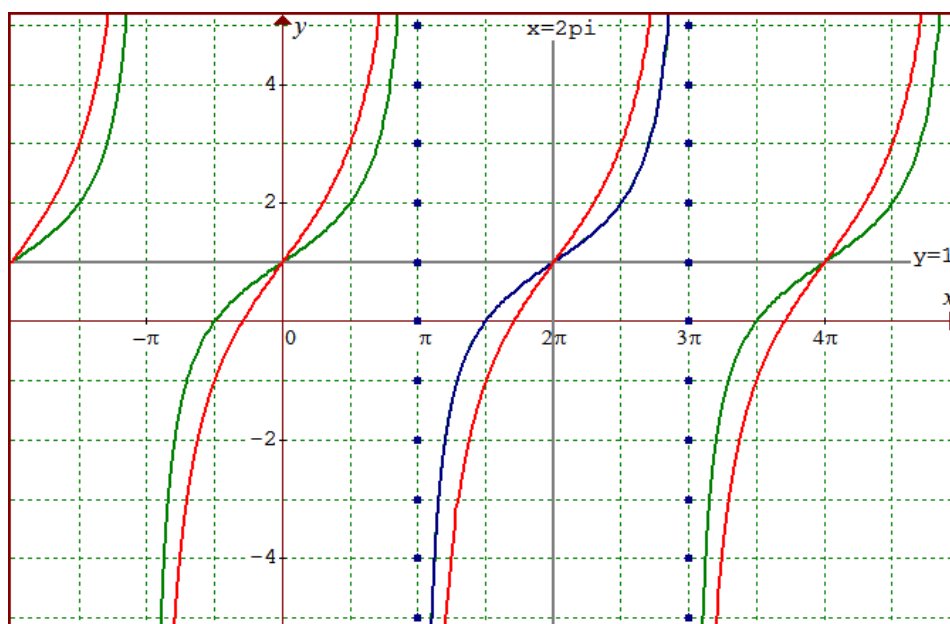
One-fourth of the cycle will have a y-value of -1 (There is no "a" value to multiply it) relative to the new axes. At three-fourths of the cycle there will be a y-value of 1 relative to the new axes.

The first cycle is in blue, with dotted line asymptotes.



Example 2: The first example had a value for “a” of 1, and so it was understood and not written. The tangent graph does not have amplitude. What affect does a different value of “a” have on the graph? That is, what if the equation in the previous example was $y = 2 \tan(.5x - \pi) + 1$? What difference is there in the graph?

While there is no amplitude, the equation simply says that y is now twice (relative to the new axes) what we had previously for each x. On the new axes we multiply each y by 2 to get the points, noting that $2(0)$ is still 0. The overall effect is still to pull on the graph vertically or stretch it vertically. This graph is in red below.



➔ Now, work the problem set for Graphing the Tangent Function.

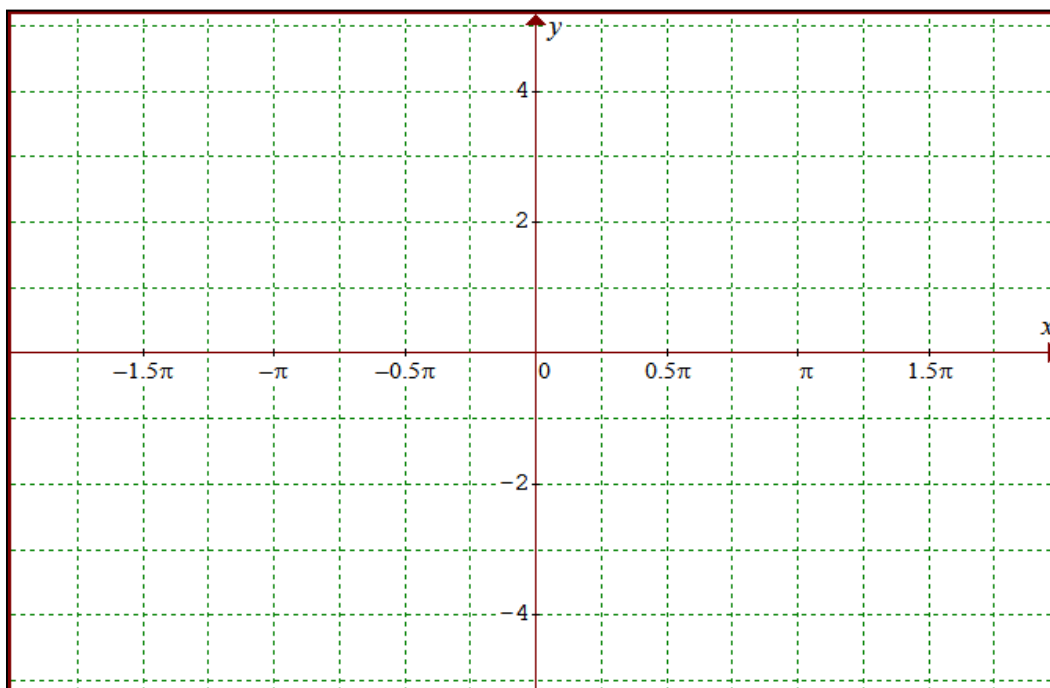
Problems for Graphing the Tangent Function

1. Find the period, vertical displacement, and asymptotes for each of the following.

a. $y = 3 \tan(2x - \pi) - 1$

b. $y = \tan\left(\frac{1}{4}x + \frac{\pi}{2}\right) + 3$

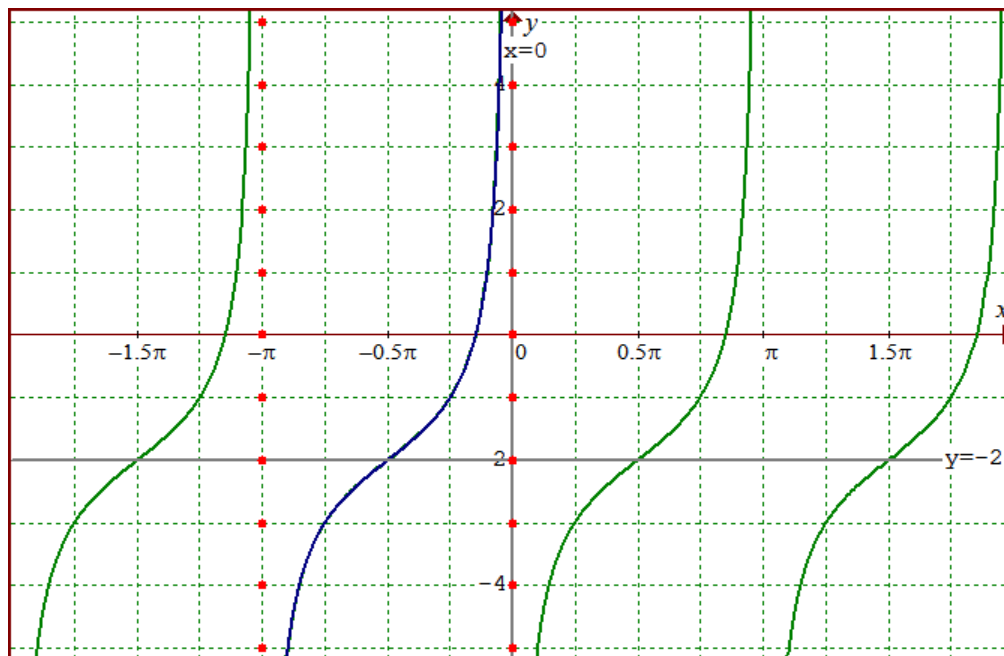
2. Find the period, vertical displacement, and equations of the asymptotes for one period for $y = \tan\left(x + \frac{\pi}{2}\right) - 2$. Then graph at least one period on the grid below. Indicate the asymptotes and new axes with dotted lines.



Answers to Problems for Graphing the Tangent Function

1. a. Period is $\frac{\pi}{b} = \frac{\pi}{2}$; vertical displacement is down 1; left asymptote found by solving $2x - \pi = -\frac{\pi}{2} \Rightarrow x = \frac{\pi}{4}$; right asymptote found by adding the period to this, so $x = \frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4}$. (Note: you can also solve $2x - \pi = \frac{\pi}{2}$ to get the right asymptote.)
- b. Period is $\frac{\pi}{b} = \pi \div \frac{1}{4} = 4\pi$; vertical displacement is up 3; left asymptote found by solving $\frac{1}{4}x + \frac{\pi}{2} = -\frac{\pi}{2} \Rightarrow x = -4\pi$; right asymptote found by adding the period to this, so $x = -4\pi + 4\pi = 0$.
2. The period is $\frac{\pi}{b} = \frac{\pi}{1} = \pi$.
 The vertical displacement is down 2. Then $y = -2$ is the new x-axis.
 The left asymptote is found by solving $x + \frac{\pi}{2} = -\frac{\pi}{2} \Rightarrow x = -\pi$.
 The right asymptote is found by adding the period to the left asymptote, $x = -\pi + \pi = 0$.
 So, $x = 0$ is the right asymptote.

Draw the asymptotes with dotted lines. Draw the new x-axis with a dotted line. Halfway through the cycle on the new x-axis is the intercept. At one-fourth of the cycle is the point with a y of -1 relative to the new x-axis. At three-fourths of the cycle is the point with a y of 1 relative to the new x-axis.



Graphing the Reciprocal Functions

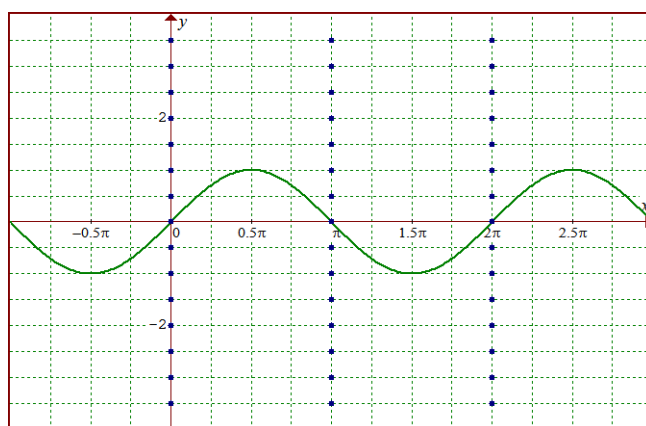
These functions are not often used, so this will be just enough to get the idea of these functions.

Remember these functions from the first week? They are as follows:

$$\csc x = \frac{1}{\sin x} \quad \sec x = \frac{1}{\cos x} \quad \cot x = \frac{1}{\tan x}$$

To better see how we get the graph of each, we will first graph the related function.

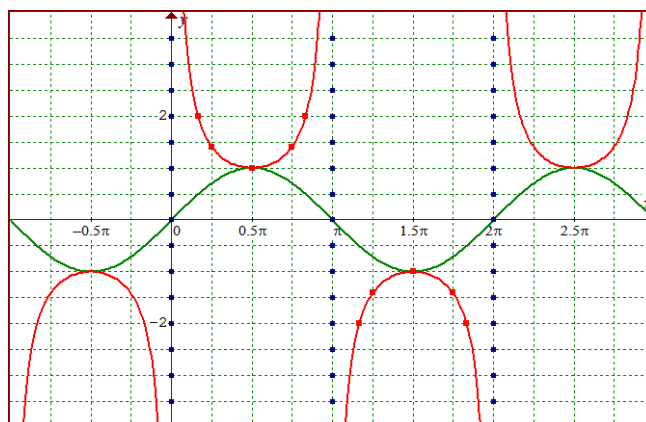
For the graph of the cosecant function, we first graph $y = \sin x$. Since $y = \csc x = \frac{1}{\sin x}$, we need to first notice that wherever $\sin x$ is 0, the cosecant will be undefined. This happens at all multiples of π , which we can write as $k\pi$, with k an integer. We will put dotted lines there.



Now we will use a calculator and find a few values given in the table below.

x	$\pi/6$	$\pi/4$	$\pi/2$	$3\pi/4$	$5\pi/6$	$-7\pi/6$	$5\pi/4$	$3\pi/2$	$7\pi/4$	$11\pi/6$
Sin x	.5	.7	1	.7	.5	-.5	-.7	-1	-.7	-.5
Csc x	2	1.4	1	1.4	2	-2	-1.4	-1	-1.4	-2

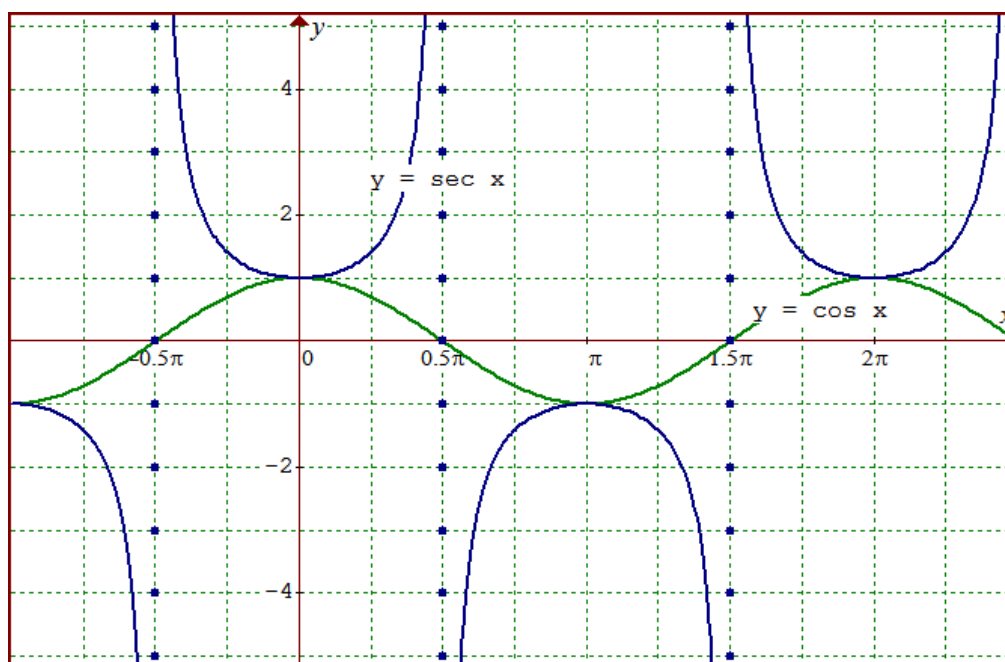
Plot the points and connect these to get a sketch. Then repeat.



The cosecant will have the same period as the sine. It is 2π . The domain is all real numbers such that $x \neq k\pi$. The range is all y such that $y \geq 1$ or $y \leq -1$. In interval notation, this is $(-\infty, -1] \cup [1, +\infty)$.

The cosecant function may involve the same transformations and translations as the other functions. While there is no amplitude, the graph of the cosecant is affected by a value that multiplies the function. For example, $y = 2 \sin x$ has an amplitude of 2, but $y = 2 \csc x$ does not have amplitude. However, the low and high turning points of each part of the curve in the graph are changed accordingly.

You should have noticed that the cosine curve is a shifted sine curve. Then $y = \sec x$ will be a shifted cosecant graph. Both $y = \cos x$ and $y = \sec x$ are graphed together as follows. It is easier to understand how the secant graph results by looking at the cosine graph, since $y = \sec x = \frac{1}{\cos x}$. The cosine graph acts as a guide somewhat, just as the sine curve acts as a guide for the cosecant graph.



We observe the following:

The domain of the secant is all real numbers such that $x \neq \frac{\pi}{2} + k\pi$, where k is an integer.

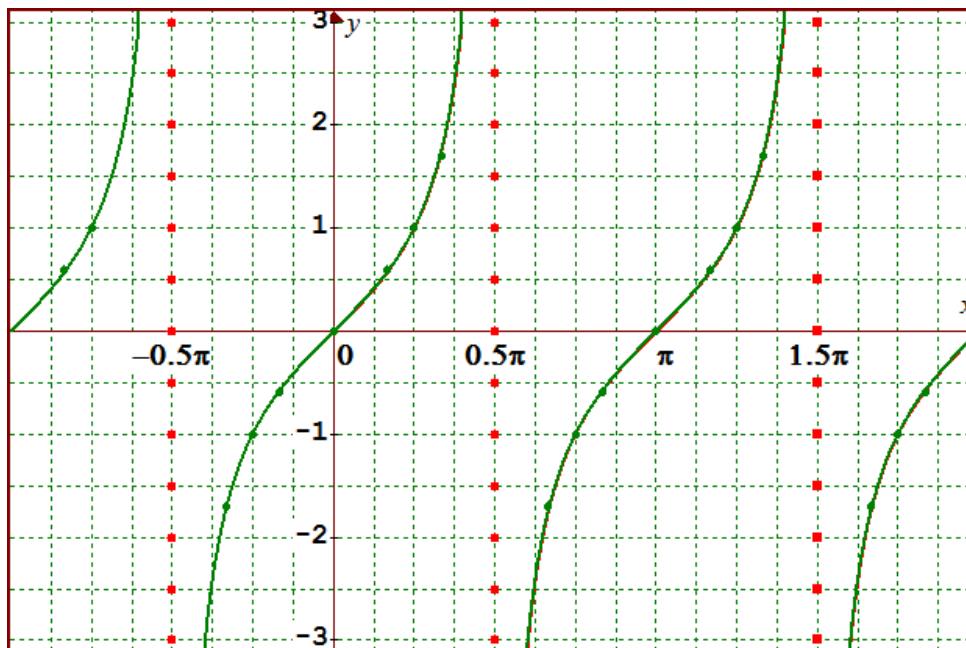
The range is all real numbers so that $y \geq 1$ or $y \leq -1$.

The period is 2π .

➔ Now work the problem set for Graphing the Reciprocal Functions.

Problems for Graphing the Reciprocal Functions

1. Recall the tangent function that we earlier studied.



Write the domain, range, and period.

2. Complete the table below by using $y = \cot x = \frac{1}{\tan x}$. One decimal place will be sufficient. Remember that when $\tan x$ is undefined, it was because the denominator was 0. A reciprocal will put 0 in the numerator (and not in the denominator).

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
Tan x	0	.6	1	1.7	Undef.	-1.7	-1	-.6	0
Cot x									

3. Plot the points on the grid in #1. Sketch a curve. Repeat as the grid allows.
4. Write the domain, range, and period for $y = \cot x$.

Answers to Problems for Graphing the Reciprocal Functions

1. For $y = \tan x$:

The domain is all real numbers such that $x \neq \frac{\pi}{2} + k\pi$.

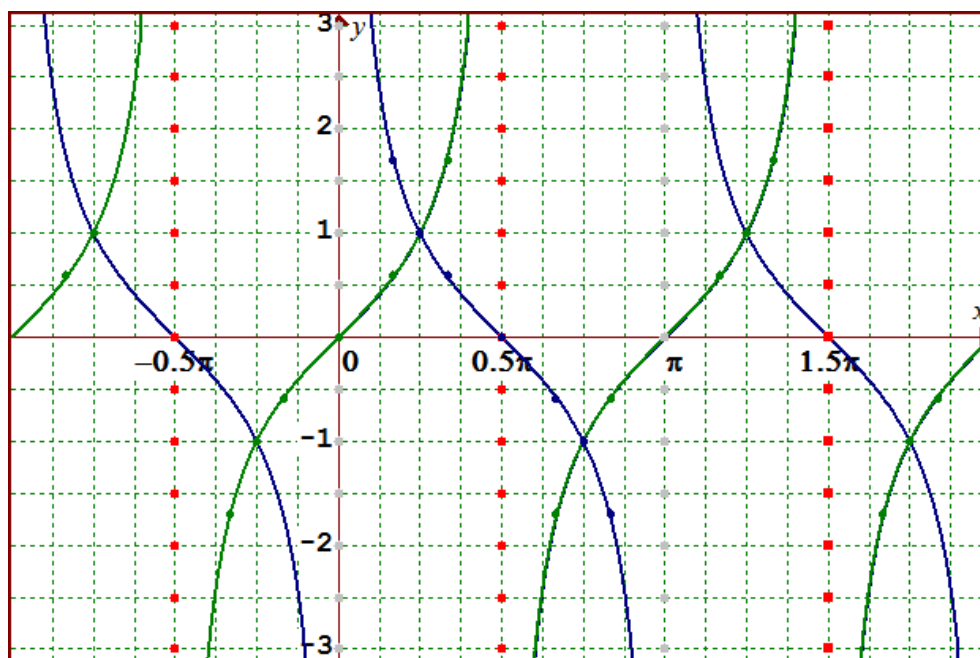
The range is all real numbers.

The period is π .

- 2.

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
Tan x	0	.6	1	1.7	Undef.	-1.7	-1	-.6	0
Cot x	Undef.	1.7	1	.6	0	-.6	-1	-1.7	Undef.

3. The cotangent function is in blue, with asymptotes as light gray dotted lines.



Asymptotes: $x = 0$

$x = \pi$

4. For $y = \cot x$:

The domain is all real numbers such that $x \neq k\pi$.

The range is all real numbers.

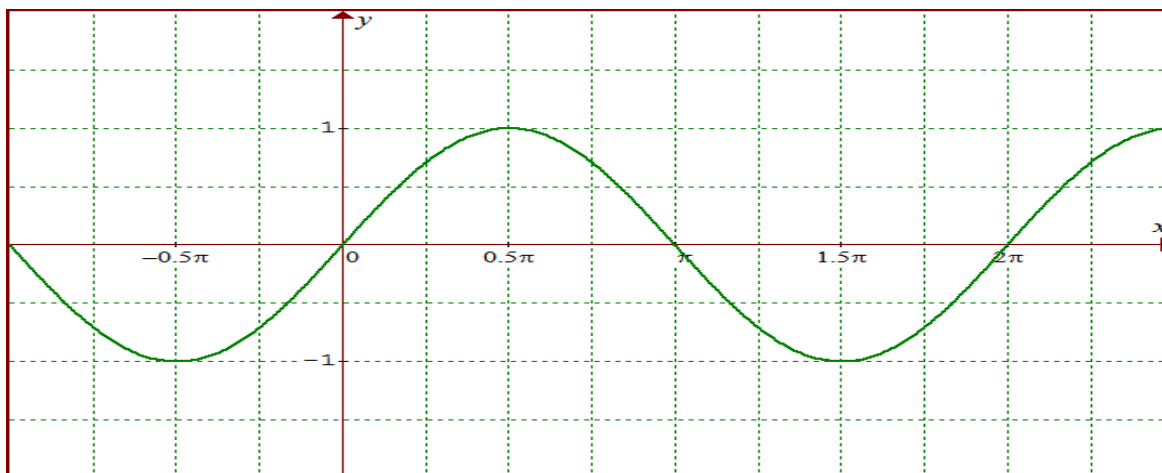
The period is π .

The Inverse Sine Function

Before talking about the inverse sine function, it is important to be sure you understand the idea of an inverse function. The following are important points.

- A function is a relation such that each x in the domain is matched with one y in the range. The graph passes the vertical line test (To show each x produces one y , the vertical line intersects the graph at one point for each x).
- A one-to-one function is one in which each y in the range comes from exactly one x in the domain. The graph also passes the horizontal line test (To show that each y comes from exactly one x , the line intersects the graph at one point for each y).
- A true inverse function is found by interchanging the x and y and solving for y in terms of x . If done successfully, the original function's domain and range are interchanged to produce the new domain and range for the inverse. This happens when the original function is one-to-one.

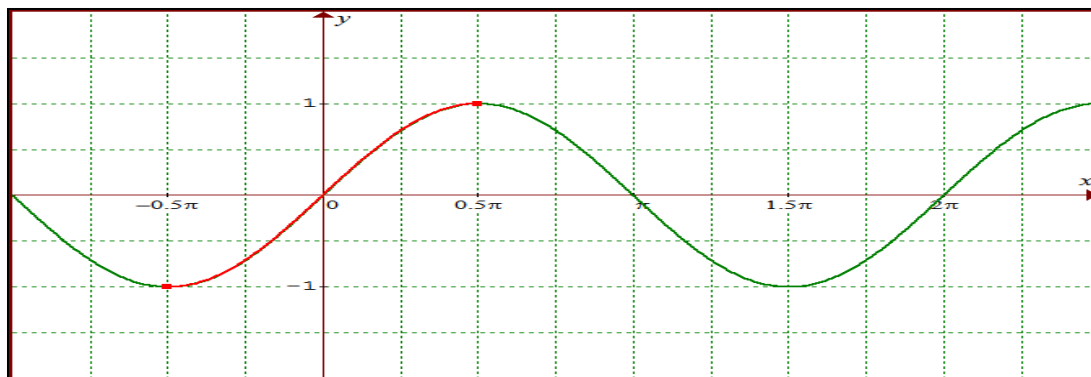
Look at the graph of the sine function again.



We already know it is a function, but you can see that it passes the vertical line test.

When we use the horizontal line test, we see that the function easily intersects the graph in more than one point for a particular y -value. For example, $y = .5$ results from $\pi/6$, and $5\pi/6$, and more.

That means the function is not one-to-one. We must restrict the domain of the sine function to a portion or interval that will be one-to-one and still yield all of the sine values. That interval is all of the real numbers in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$. Another way of writing this is $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. We mark that part of the graph and focus on it.



With x in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$ for the domain, we have y in the interval $[-1, 1]$ for the range.

To find the inverse we interchange x and y in the function $y = \sin x$ to get $x = \sin y$. Usually at this point there is some algebra we can use to solve for y , but there is none for this simple equation. We do, however, want the emphasis on y . That is, we want $y = \underline{\hspace{2cm}}$. We have x as the sine value now, so y is the real number or angle that has x as the sine.

The function gets a new name and notation:

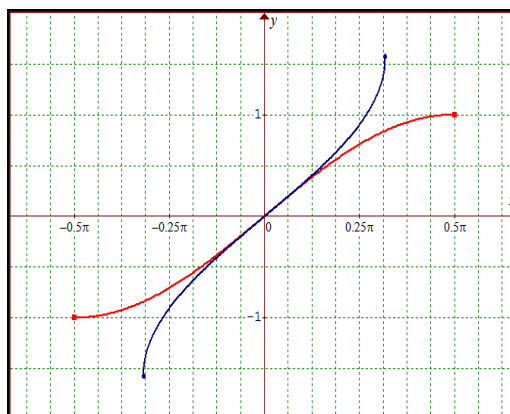
$$y = \sin^{-1} x \text{ or } y = \arcsin x.$$

The first notation we can simply read as “ y is the inverse sine of x ”. Be careful with this notation.

The “-1” written in exponent position is not an exponent. That is, $\sin^{-1} x \neq \frac{1}{\sin x}$.

The second notation we read as “ y is the arc whose sine is x ”. Why do we use “arc”? Remember that we used t as a real number matched to the length of an arc on the unit circle and we developed functions of t . In some circumstances x might be an angle, so you can say “ y is the angle whose sine is x ”.

What does the inverse function look like when we graph it? The blue graph is $y = \sin^{-1} x$. The domain is $[-1, 1]$ and the range is $[-\frac{\pi}{2}, \frac{\pi}{2}]$. (The x -axis is in terms of π , e.g., $.25\pi \approx .8$, $.5\pi \approx 1.6$, etc.)



Example 1: Find y , or evaluate, if $y = \sin^{-1}\left(\frac{1}{2}\right)$.

We read and think “ y is the number or arc whose sine is $\frac{1}{2}$ ”. We recognize that as a **special** sign value of $\frac{\pi}{6}$, which is in the range of the inverse sine. If our situation called for angles, we would answer that y is 30° .

Example 2: Evaluate $y = \arcsin\left(-\frac{\sqrt{2}}{2}\right)$.

We read and think “ y is the arc whose sine is $-\frac{\sqrt{2}}{2}$ ”. We know the **reference** number for the special sine value of $\frac{\sqrt{2}}{2}$ is $\frac{\pi}{4}$. Since the sine here is negative, the number in the range of the inverse function is in the fourth quadrant and is $-\frac{\pi}{4}$. If our situation called for angles, we would answer that y is -45° .

We have to think in a different way here. While we must evaluate these expressions limited to the domain and range of the inverse function, these first answers may not be the actual answer to a problem.

In Example 1, the inverse function limits our answer to $\frac{\pi}{6}$ or 30° . In a particular problem, however, we may really need to find an obtuse angle to meet the requirements of the problem. Then we have to find the angle in the second quadrant that uses 30° as reference. That would be 150° . In terms of real numbers, it is $\frac{5\pi}{6}$.

We use the inverse function to find the actual number or angle, or else its reference number or angle. The final answer depends on the problem.

Example 3: Find $\sin^{-1}(-.1234)$ to four decimal places.

This sine value is certainly nothing we recognize. This is where your calculator helps. They are programmed to give answers based on inverse functions.

For graphing calculators, enter the expression as you read it. The “ \sin^{-1} ” should be a secondary function with “ \sin ” on the top of the key. Enter the following, in radian mode:

$$\boxed{2^{\text{nd}}}\ \boxed{\sin}\ (\ \boxed{-}\)\ .1234\ \boxed{\text{enter}} = -.123715 \text{ or } -.1237$$

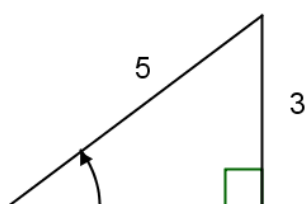
$$\text{So } \sin^{-1}(-.1234) = -.1237.$$

If we need an angle in degrees, in degree mode the calculator yields -7.0884° . (You write the “ $^\circ$ ”.) That may have to be rewritten as a positive angle, depending on the problem.

The next example involves composition of functions. I won't go into detail here, but briefly explain the idea. One function is performed on an element in its domain. The result is then used to perform another function.

Example 4: Evaluate $\cos\left(\arcsin\frac{3}{5}\right)$.

We read this as “the cosine of the arc whose sine is $\frac{3}{5}$ ”. Since we just want the cosine and not the arc (real number or angle), we do not have to have the arc. We know that the sine is $\frac{3}{5}$. Draw a right triangle for reference. We need to find the other side. This should be very familiar.



$$\begin{aligned} a^2 + b^2 &= c^2 \\ 3^2 + b^2 &= 5^2 \\ b^2 &= 16 \\ b &= 4 \end{aligned}$$

Therefore, the cosine is $\frac{4}{5}$. We have $\cos\left(\arcsin\frac{3}{5}\right) = \frac{4}{5}$.

If you recognize the function given as the value of a special angle or number, you need not go through this.

Problems may use a variety of variable names. Don't become dependent on using x or any particular variable name. Think about what you are doing and what is required to do it.

➔ **Now work the problem set for the Inverse Sine Function.**

Problems for the Inverse Sine Function

1. Find, or evaluate, each of the following exactly.

a. $\sin^{-1}\left(-\frac{1}{2}\right)$

b. $\arcsin\left(\frac{\sqrt{3}}{2}\right)$

c. $\arcsin\left(\frac{\sqrt{2}}{2}\right)$

d. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

2. Use your calculator to find each of the following, In radian mode, find to 4 decimal places. Then use degree mode and find to the nearest tenth of a degree.

a. $\arcsin(.3224)$

b. $\sin^{-1}(-.4029)$

c. $\sin^{-1}(2.3554)$

d. $\arcsin(.8614)$

e. $\arcsin(-.9005)$

3. Find each of the following exactly.

a. $\cos\left(\arcsin\left(\frac{4}{5}\right)\right)$

b. $\sin(\arcsin(.4532))$

Answers to Problems for the Inverse Sine Function

1.

- a. We recognize the special sine value for $\frac{\pi}{6}$ (or 30°) as the reference. Since the sine is negative, $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$ (or -30°).
- b. We recognize the special sine value for $\frac{\pi}{3}$ (or 60°) as the reference. So $\arcsin\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$.
- c. We recognize the special sine value for $\frac{\pi}{4}$ (or 45°) as the reference. Then we have $\arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$.
- d. We recognize the sine value for $\frac{\pi}{3}$ (or 60°) as the reference. Since the sine is negative, then $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$.

2.

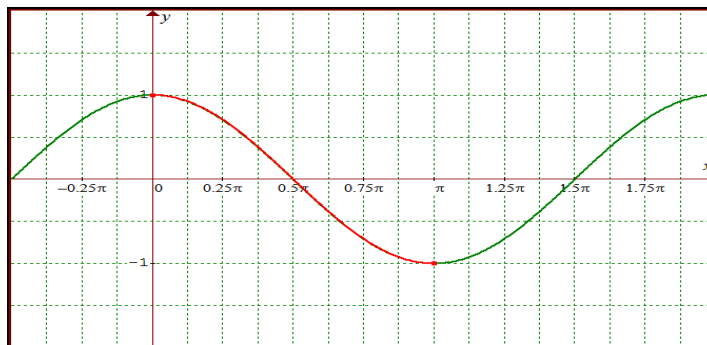
- a. $\arcsin(.3224) = .3282$ or 18.8°
- b. $\sin^{-1}(-.4029) = -.4147$ or -23.8°
- c. $\sin^{-1}(2.3554)$ does not exist, since 2.3554 is not in the domain.
- d. $\arcsin(.8614) = 1.0380$ or 59.5°
- e. $\arcsin(-.9005) = -1.1209$ or -64.2°

3.

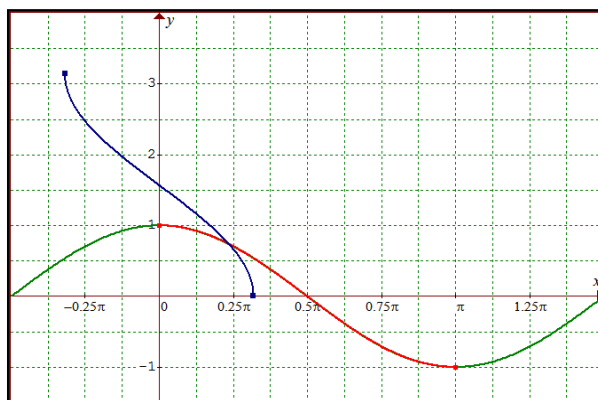
- a. $\cos\left(\arcsin\left(\frac{4}{5}\right)\right) = \frac{3}{5}$, which is worked just like Example 4.
- b. $\sin(\arcsin(.4532)) = .4532$, since this reads as "the sine of the arc whose sine is .4532".

The Inverse Cosine and Inverse Tangent Functions

We do the same thing to arrive at the inverse cosine function as we did for the inverse sine. We find a one-to-one portion of the graph that includes all of the cosine values. See the graph below.

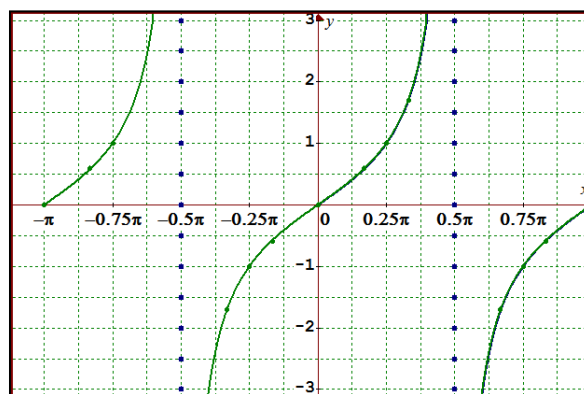


This restricts the domain to the interval $[0, \pi]$ and the range is $[-1, 1]$. We interchange x and y in the equation for the cosine of $y = \cos x$ to get $x = \cos y$. We write that $y = \cos^{-1} x$ or $y = \arccos x$.

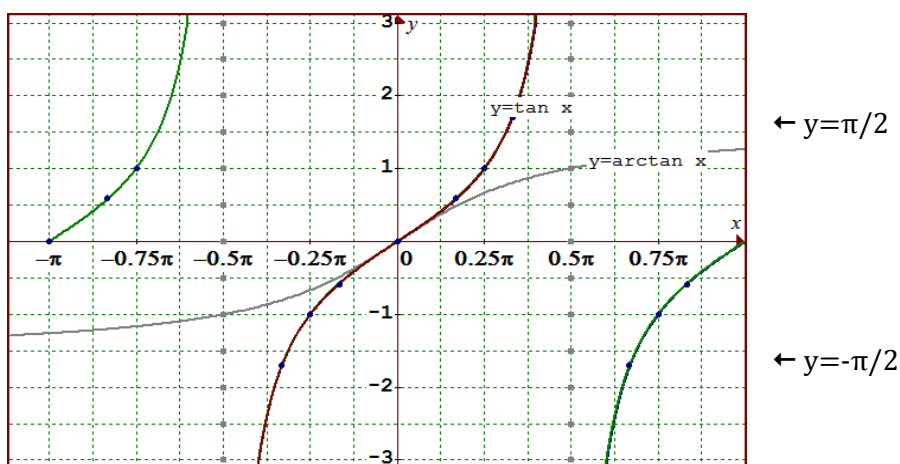


We see that the domain of the inverse cosine is now $[-1, 1]$ and the range is $[0, \pi]$. The x -axis is still in terms of π and $.25\pi$ is about $.8$.

Now we take a look at the tangent function again.



The first cycle is one-to-one and takes on all of the tangent values. We will use that restricted domain of $(-\frac{\pi}{2}, \frac{\pi}{2})$ and the range of all real numbers or $(-\infty, \infty)$. We interchange x and y in the equation $y = \tan x$ to get $x = \tan^{-1} y$. We write $y = \tan^{-1} x$ or $y = \arctan x$.



The domain for the inverse function is now $(-\infty, \infty)$ and the range is $(-\frac{\pi}{2}, \frac{\pi}{2})$ with horizontal asymptotes at $y = \pi/2$ and $y = -\pi/2$.

We work with these two inverse functions the same way we worked with the inverse sine.

Example 1: Find the exact value for each of the following.

- $\cos^{-1}(-1)$
- $\arctan(0)$

For part a: We read “the arc or angle whose cosine is -1”. We recognize that as the cosine of π or 180° . Therefore, $\cos^{-1}(-1) = \pi$ (or 180°).

For part b: We read “the arc or angle whose tangent is 0”. We recognize 0 as the tangent of 0. Therefore, $\arctan(0) = 0$.

Example 2: Use your calculator to find each of the following, to 4 decimal places in radian mode, and to the nearest tenth of a degree in degree mode.

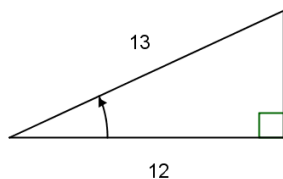
- $\arccos(.7865)$
- $\tan^{-1}(-12)$

For part a: $\arccos(.7865) = .6657$ or 38.2°

For part b: $\tan^{-1}(-12) = -1.4877$ or -85.2°

Example 3: Find the exact value of $\tan\left(\cos^{-1}\left(-\frac{12}{13}\right)\right)$.

We read this as “the tangent of the arc or angle whose cosine is $-\frac{12}{13}$ ”. Since we really just want the tangent and we are given the cosine, we do not need to know the angle. We draw a triangle. We need to find the other side in order to get the tangent value.



$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 12^2 &= 13^2 \\ a^2 &= 25 \\ a &= 5 \end{aligned}$$

The tangent for our angle is $\frac{5}{12}$. However, we must look at the fact that the cosine was negative, which means a second quadrant angle or a number between $\frac{\pi}{2}$ and π as indicated by the range of the inverse cosine. In the second quadrant, the tangent is negative. So $\tan\left(\cos^{-1}\left(-\frac{12}{13}\right)\right) = -\frac{5}{12}$.

Composition of functions can require a lot of careful thinking. There may not be a lot of problems on a test, but you need to practice some, just in case.

I will not cover the other three inverse functions because they are not often used. See your textbook if you need them.

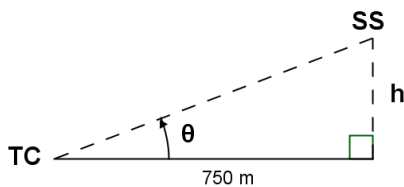
Why should we bother with inverse functions?

In algebra, we are able to solve an equation and get to that final step that says $x = 5$ or some other number. In trigonometric equations, we will get down to a step that says something like $\sin x = .5$. We need the inverse function to finish so that we can solve for x , for example $x = \sin^{-1}.5$.

➔ **Now do the problem set for The Inverse Cosine and Inverse Tangent Functions.**

Problems for The Inverse Cosine and Inverse Tangent Functions

- Find the exact value for each of the following.
 - $\arccos\left(\frac{\sqrt{2}}{2}\right)$
 - $\tan^{-1}(-1)$
 - $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
- Use your calculator to find each of the following, to 4 decimal places in radian mode, and the nearest tenth of a degree in degree mode.
 - $\arctan(-8.4625)$
 - $\cos^{-1}(.6112)$
- A television camera at ground level is filming the lift-off of a space shuttle at a point 750 meters from the launch pad. Let θ be the angle of elevation to the shuttle and let h be the height of the shuttle.



- Write θ as a function of h . (First, use a trigonometric function for acute angles in a right triangle.)
- Find θ to the nearest tenth of a degree when $h = 1200$ meters.

Answers to Problems for the Inverse Cosine and Inverse Tangent Functions

1.

- a. The problem $\arccos\left(\frac{\sqrt{2}}{2}\right)$ reads as “the arc or angle whose cosine is $\frac{\sqrt{2}}{2}$ ”. We recognize this special cosine as that of $\frac{\pi}{4}$ (or 45°). Since $\frac{\pi}{4}$ is in the range of the inverse cosine, $\arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$.
- b. The problem $\tan^{-1}(-1)$ reads as “the arc whose tangent is -1”. We recognize the value of 1 as the tangent of $\frac{\pi}{4}$ (or 45°). The tangent is negative in the second and fourth quadrants. But the inverse tangent has values in the fourth quadrant. However, they are written as a negative arc or number. Therefore, $\tan^{-1}(-1) = -\frac{\pi}{4}$.
- c. This problem is read as “the arc or angle whose cosine is $-\frac{\sqrt{3}}{2}$ ”. We recognize $\frac{\sqrt{3}}{2}$ as the cosine of $\frac{\pi}{6}$ or 30° . The cosine is negative in the second and third quadrants. But the inverse cosine uses the second quadrant. Therefore, $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$ (or 150°).

2.

- a. $\arctan(-8.4625) = -1.4532$ or -83.3°
- b. $\cos^{-1}(.6112) = .9132$ or 52.3°

3. From the diagram, we can write a tangent function.

- a. $\tan \theta = \left(\frac{h}{750}\right)$. Now we write the inverse function to get $\theta = \tan^{-1}\left(\frac{h}{750}\right)$.
- b. For a height of 1200 meters, we evaluate $\theta = \tan^{-1}\left(\frac{1200}{750}\right)$. For this problem, use degree mode on your calculator: $\boxed{\tan^{-1}} \boxed{[$ 1200 $\boxed{]} \boxed{\div}$ 750 $\boxed{]} \boxed{\text{enter}}$.

To the nearest tenth of a degree, $\theta = 58.0^\circ$ (from 57.99).

Note. In word problems or applications, should you get a negative answer, make sure it makes sense in the words of the problem or meets the requirements of the problem. You may need to convert to an equivalent coterminal positive number or angle.